# The Observability Function

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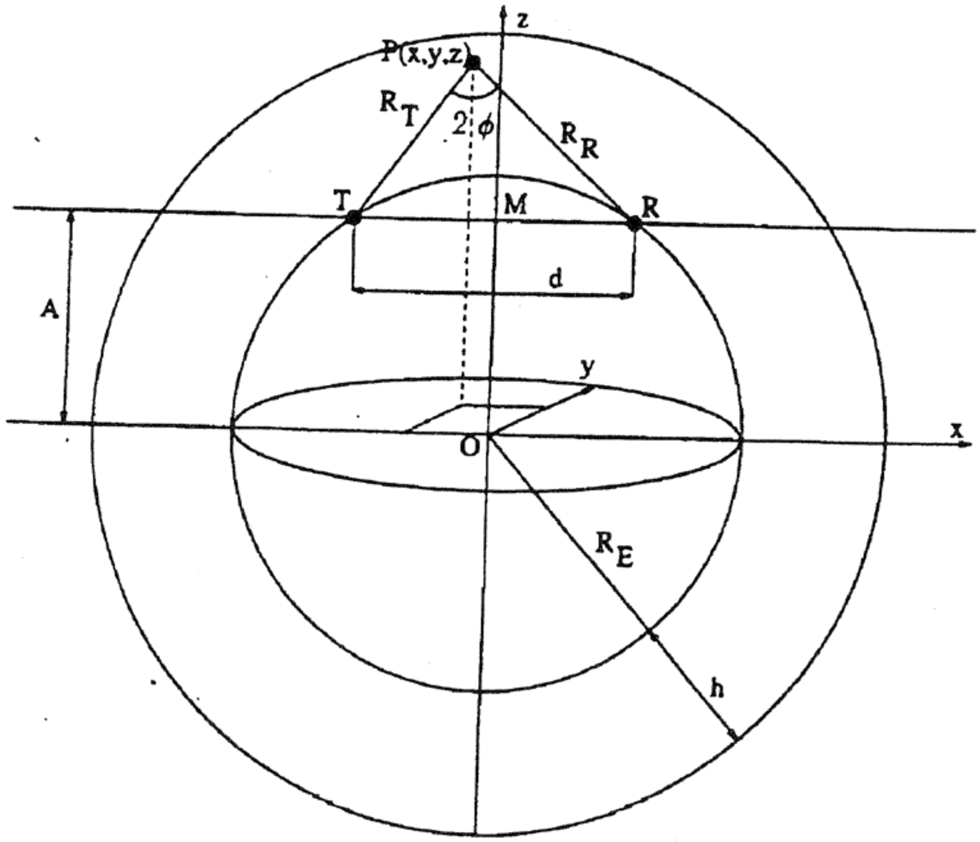
- The **Observability Function (OF)** tries to estimate the **sensitivity of the setup for underdense meteors** from a particular meteor stream at a particular time and location (i.e., with a particular radiant position)
- The **flux density** provides an **absolute measure** expressed in **number of particles per 1000 km^2 per hour** (larger than some reference mass, e.g., 10^-3 g). It is more difficult to calculate than the OF.
- The **OF** provides a **relative measure**. So **N_corr = N_obs / OF** should be C * number of particles per 1000 km^2 per hour, but we don’t know the value of the constant C.
- The calculation of the OF and the flux density both require as **input** the **complete gain patterns of the transmitting and receiving antennas** (measured or modeled) and the **effective threshold level of the setup**.




- We consider a **transmitter T** and **receiver R** at distance d, and a coordinate system as shown in the figure above.
- We consider **reflection points P(x,y,z)** which lies at a height h (100 km) above the Earth's surface.
- We consider a **radiant direction (xi, eta, zeta)** and consider a meteor trail with that direction at the reflection point.
- **A point is a reflection point if and only if the travel path of the radio wave from T to P to R is minimal**. This corresponds to a **geometrical condition which we can calculate** (this means the meteor trail is the tangent to an ellipsoid of revolution with foci in T and R).
- To this condition, we add the condition that the reflection point is at height h. This leads to a 6-th order polynomial equation in the coordinates x and y of the reflection point. This polynomial equation is solved numerically, and the solutions are **potential reflection points**, i.e., points in which the meteor trail has the right orientation to reflect radio waves from T to R.
- The collection of potential reflection points for a particular distance between T and R and radiant direction is a curve. In case of radar, this curve is a great circle in the sky. Here are some examples of such curves:

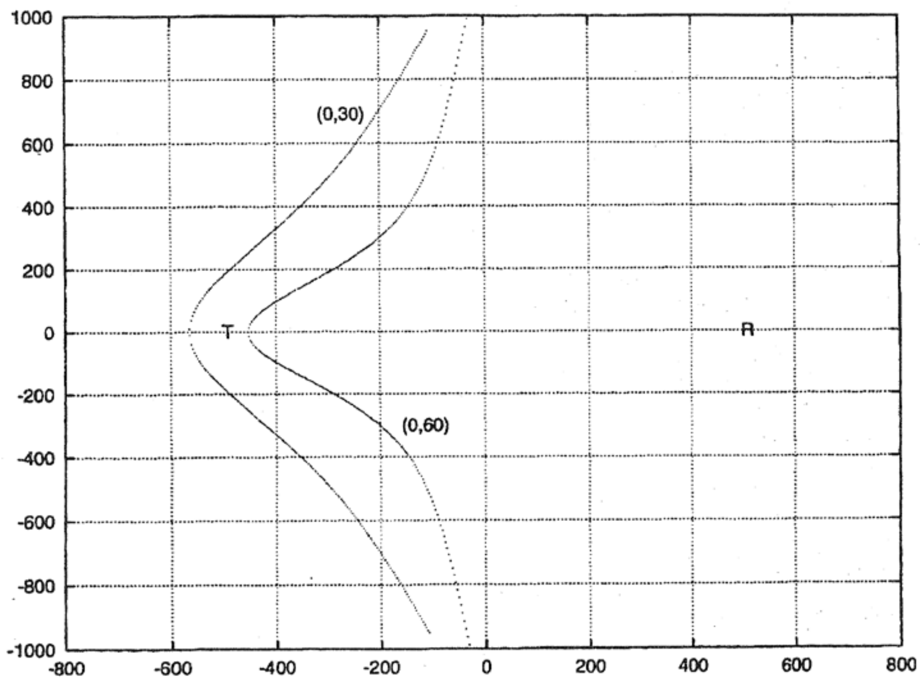



Figure 4: The $(s = 0)$-curve for $d = 1000$ km, $h = 100$ km, for the radiant positions $\varphi = 0°$, $\theta = 30°$ and $\varphi = 0°$, $\theta = 60°$.

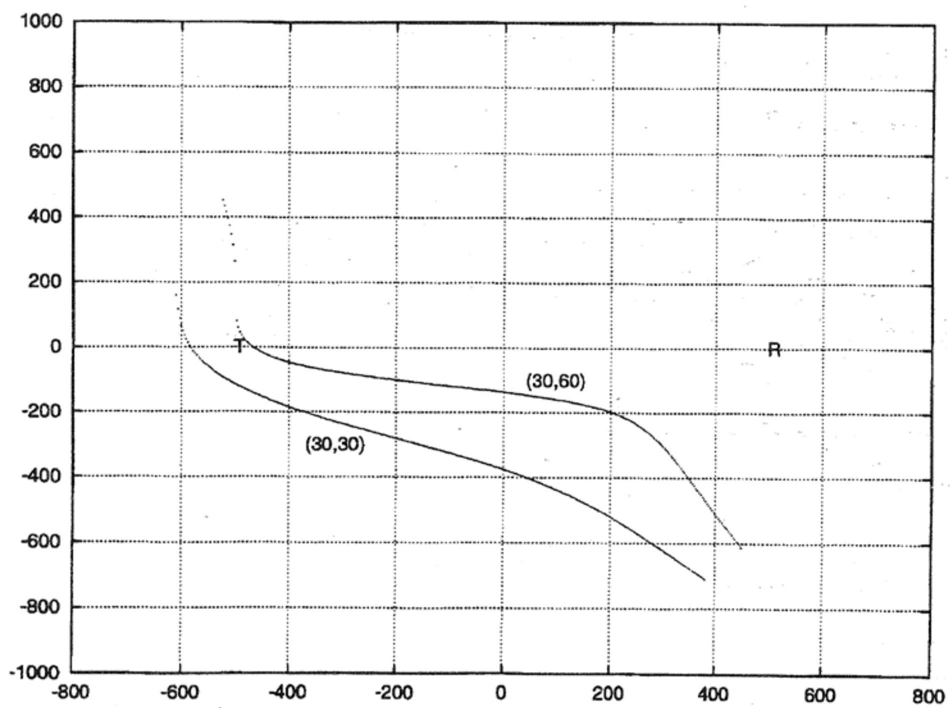



Figure 5: The $(s = 0)$-curve for $d = 1000$ km, $h = 100$ km, for the radiant positions $\varphi = 30°$, $\theta = 30°$ and $\varphi = 30°$, $\theta = 60°$.

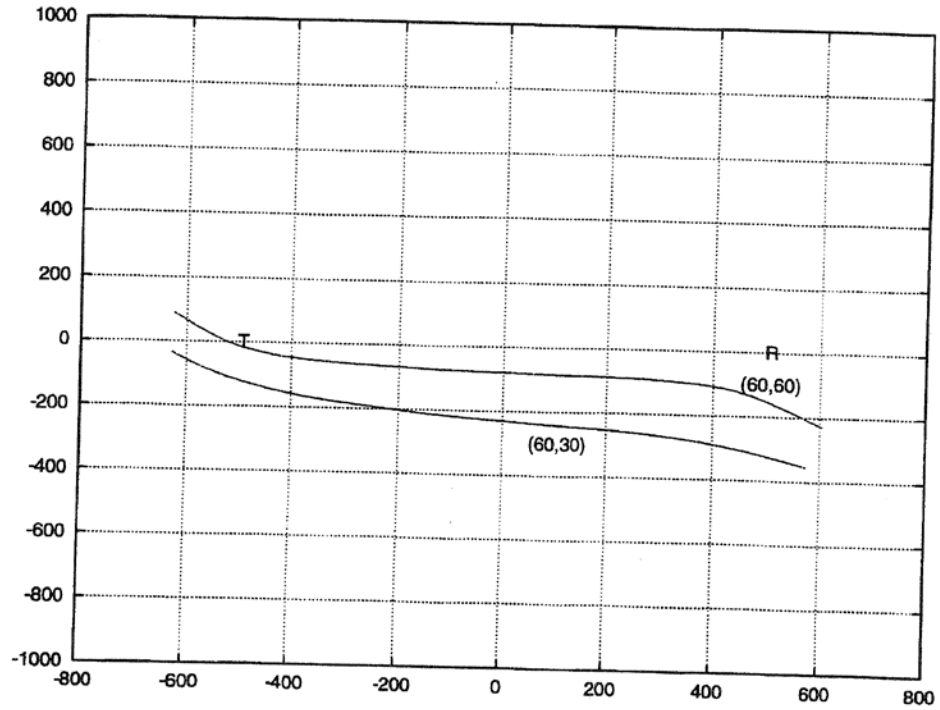



Figure 6: The $(s = 0)$-curve for $d = 1000$ km, $h = 100$ km, for the radiant positions $\varphi = 60°$, $\theta = 30°$ and $\varphi = 60°$, $\theta = 60°$.

- If radio waves are reflected from T to R in a potential reflection point, **we will only detect the signal if it lies above the signal detection threshold of our receiver.**
- Using physical considerations, for any potential reflection point we can **calculate the power P_R received in the receiver, starting from the power P_T transmitted by the transmitter**. It is a complicated formula, which contains o.a. the transmitter and receiver gains and the electron line density alpha at the reflection point.
- Using the mass index s of the meteor stream and the assumption that the masses in the meteor stream are distributed as a power law, we can estimate the probability that an underdense shower meteor appearing the potential reflection point P would generate a power P_T in the transmitter that exceeds the signal detection threshold of our receiver. This is the **probability that for this potential reflection point, we will detect the signal in the receiver**.
- **We calculate this probability for all points on the potential reflection curve**, and add the probabilities (i.e., we integrate along the curve). **This is the OF**.
- Cis wrote a C++ program to calculate the OF, nearly finished. Hervé is writing a Matlab version.